Wide-Band Polarimetric Radar Inversion
Studies for Vegetation Layers

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Abstract—In this paper, we show how the entropy-alpha target decomposition scheme may be used for parametric inversion studies on random particle cloud models for vegetation layers. The decomposition is detailed first and then applied to a two-parameter model for backscatter from a random cloud of small anisotropic particles. The two main parameters used are the mean particle shape and the mean orientation angle of the cloud. An inversion algorithm is presented and applied to broad-band polarimetric radar data from the European Microwave Signature Laboratory (EMSL), Ispra, Italy. The results have been obtained from measurements of a fir tree and a ficus tree. They show a wavelength scale dependence of the shape and distribution of scatterers, which reflects the complex volume scattering nature of such problems. Moreover, the values and trends from these two trees as a function of the frequency are different, as expected from their physical structures. Consequently, this algorithm has the potential to be useful in the construction of classification schemes for vegetation.

Index Terms—Inverse problems, radar polarimetry, synthetic aperture radar (SAR).

I. INTRODUCTION

For some time, there has been a great deal of interest in the use of radar polarimetry for remote sensing of forest and other vegetated terrain [1]–[5]. All such techniques are based on the fact that radar scattering from natural terrain cover is strongly anisotropic and hence, it leads to changes in polarization due to scattering. In the quantitative exploitation of such effects, the choice of radar frequency and angle of incidence is critical [6], [7]. Furthermore, the stochastic nature of such scattering problems demands averaging the received signal and subsequently extracting biophysical parameters from backscatter amplitude or phase measurements [8].

In this paper, we consider the effect radar frequency has on the polarization behavior of backscatter from forests. Although only a single tree was used in each of the experiments, the averaging required can be carried out by observing the tree over a range of angles in azimuth and elevation. In this way, we make use of a three-dimensional (3-D) data set (azimuth, elevation, and frequency being the three parameters varied). The only physical component missing from these experiments is the effect of the ground and ground–tree interactions. We then employ a random particle cloud model to interpret the data.

We further show how an inversion of the experimental data may be used to obtain three important physical parameters: the mean particle shape and the mean and spread of particle orientation angle from the averaged polarimetric response.

In Section II, we review target decomposition theory as it is used to interpret the averages generated from the experimental data set. In Section III, we outline the particle cloud model employed and show how it may be mapped into the target decomposition formulation. In Section IV, we describe the experimental configuration and interpret the fine structure of the data set through 3-D synthetic aperture radar (SAR) imaging techniques. Finally, in Section V, we employ averages over the data set to simulate radar measurement scenarios and employ the averages in an inversion scheme to extract important physical parameters.

II. TARGET DECOMPOSITION THEORY

A. Coherent Decomposition

In its original form, the Pauli matrix decomposition consisted of a coherent sum of up to four scattering mechanisms (isotropic surface and dihedral scattering and a cross-polarizer). Note that in this context, isotropic means that $[HH] = [VV]$. Equation (1) summarizes the structure of this original decomposition verbally and in terms of the Pauli matrices [9]

$$[S] = \begin{bmatrix} a+b & c+i\theta & 0 \\ c+i\theta & a-b & 0 \\ 0 & 0 & d \end{bmatrix}$$

$$= a\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} + b\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} + d\begin{bmatrix} 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix}.$$
written as a complex vector

\[
\mathbf{z} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = |A| \begin{bmatrix} \cos \phi e^{i \theta} \\ \sin \alpha \cos \beta e^{i \phi} \\ \sin \alpha \sin \beta e^{i \gamma} \end{bmatrix}.
\] (2)

To change from one mechanism to another, the corresponding transformation matrices for changes in \( \alpha \) and \( \beta \) are simple plane rotations as shown in (3)

\[
\begin{align*}
\mathbf{z}' &= \begin{bmatrix} 1 & 0 & 0 \\
0 & \cos \Delta \beta & -\sin \Delta \beta \\
0 & \sin \Delta \beta & \cos \Delta \beta \end{bmatrix} \mathbf{z} \\
\mathbf{z}'' &= \begin{bmatrix} \cos \Delta \alpha & -\sin \Delta \alpha & 0 \\
\sin \Delta \alpha & \cos \Delta \alpha & 0 \\
0 & 0 & 1 \end{bmatrix} \mathbf{z}'
\end{align*}
\] (3)

This observation leads us to the following important theorem.

B. Point Target Reduction

**Theorem:** Any polarimetric back-scattering mechanism obeying reciprocity can be reduced to the identity matrix by a series of three matrix transformations as shown in (4)

\[
\begin{align*}
\mathbf{z} &= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\
-\sin \alpha & \cos \alpha & 0 \\
0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\
0 & \cos \beta & \sin \beta \\
0 & -\sin \beta & \cos \beta \end{bmatrix} \\
&\quad \times \begin{bmatrix} e^{-i \phi} & 0 & 0 \\
0 & e^{-i \theta} & 0 \\
0 & 0 & e^{-i \gamma} \end{bmatrix} \mathbf{k}
\end{align*}
\] (4)

where the angles are defined as follows:

\( \alpha \) target scattering mechanism \( 0 \leq \alpha \leq 90^\circ \)

\( \beta \) target orientation \( -180^\circ \leq \beta < 180^\circ \)

\( \phi, \delta, \gamma \) target phase angles

An important observation following from the algebraic properties of the Pauli spin matrices is that the angle \( \beta \) in this decomposition represents the physical orientation of the scatterer about the line of sight. The angle \( \alpha \), however, is not related to target orientation, even though it appears in the mathematical form of a plane rotation in (4). It represents an internal degree of freedom of the target and can be used to describe the type of scattering mechanism as we show now.

The \( \alpha \) parameter is a continuous angle with a range of \( 90^\circ \). It can be used to represent a wide variety of scatterers, as shown in Fig. 1. At \( \alpha = 0 \), we obtain an isotropic surface (the first Pauli matrix). However, as \( \alpha \) increases, the surface becomes anisotropic (i.e., \( [HH] \neq [VV] \)). In the extreme, \( \alpha = 45^\circ \), at which point we have a dipole scattering mechanism, and \( HH \) or \( VV \) is zero (which one is zero is determined by \( \beta \)). If \( \alpha > 45^\circ \), then we obtain an anisotropic dihedral (i.e., \( [HH] \neq [VV] \) and the phase difference is \( 180^\circ \)). In the limit \( \alpha = 90^\circ \), we obtain the second Pauli matrix (and the third as well, which differs from the second only by a rotation). This point also can be used to represent targets that cause a phase shift between \( h \) and \( v \) incident wave components. In particular, the most extreme example of such a scatterer is the helix. Note that \( \alpha \) is rotation invariant (i.e., it is decoupled from \( \beta \)), so we can identify the scattering mechanism independently of its physical orientation.

To reinforce these ideas, in Table I we show a list of the parameter values for a range of canonical scattering mechanisms of interest in polarimetry (the symbol \( \infty \) means that the parameter has no fixed value for that scattering mechanism). We can see that the \( \alpha \) parameter provides a powerful extension of the basic decomposition in (1), as it frees us from having to consider only isotropic mechanisms. Since, in this sense, isotropy normally is associated with man-made calibration reflectors rather than with natural media, the importance of \( \alpha \) is that it permits us to extend the decomposition into more practical remote sensing applications.

C. Generalized Stochastic Decomposition

The second important property of (1) is the orthogonality of the elements of the decomposition. This permits us to extend the previously mentioned coherent decomposition into the partially coherent case by considering a second-order product formulation based on the vector \( \mathbf{z} \). This leads to the well-known hermitian coherency matrix, which has a 1-1 mapping with the Mueller matrix (or Stokes reflection matrix in backscatter problems). Hence, such a product contains information about depolarization effects and the variance of system noise. The key observation to make is that this second-order product is always an hermitian matrix, which can be decomposed into a real eigenvalue spectrum and orthogonal unitary eigenvectors. In general, we can write

\[
[T] = [U_3][\lambda_1 \ 0 \ 0 ; 0 \ \lambda_2 \ 0 ; 0 \ 0 \ \lambda_3][U_3]^T
\] (5)

with

\[
[U_3] = \begin{bmatrix}
\cos \alpha_1 & \cos \alpha_2 & \cos \alpha_3 \\
\sin \alpha_1 \cos \beta_1 e^{i \phi_1} & \sin \alpha_2 \cos \beta_2 e^{i \phi_2} & \sin \alpha_3 \cos \beta_3 e^{i \phi_3} \\
\sin \alpha_1 \sin \beta_1 e^{i \theta_1} & \sin \alpha_2 \sin \beta_2 e^{i \theta_2} & \sin \alpha_3 \sin \beta_3 e^{i \theta_3} \\
\end{bmatrix}
\]

where \( \lambda_1 \geq \lambda_2 \geq \lambda_3 \geq 0 \). In the case of random media with azimuthal symmetry, (5) simplifies to the case shown in (6). We can see that in this special case, \( \alpha_1 + \alpha_2 = 90^\circ \), \( \beta_1 = \beta_2 = 0 \), and \( \alpha_3 = \beta_3 = 90^\circ \). The consequence of this is...
that we need only one scattering mechanism ($\alpha$) to describe azimuthally symmetric random media, see (6), shown at the bottom of the page.

When azimuthal symmetry is not present (which will be the case when we have, for example, a cloud of particles with preferred orientation), we must resort to (5). By using a three-symbol Bernoulli model, we can define an average scattering mechanism $\bar{\alpha}$ and an average orientation angle $\bar{\beta}$ such that [10]

$$\bar{\alpha} = P_1\alpha_1 + P_2\alpha_2 + P_3\alpha_3 \quad \bar{\beta} = P_1\beta_1 + P_2\beta_2 + P_3\beta_3$$

which has an entropy or uncertainty given by $H$, such that $0 \leq H \leq 1$

$$H = -P_1 \log_2 P_1 - P_2 \log_2 P_2 - P_3 \log_2 P_3 \quad P_i = \frac{\lambda_i}{\sum \lambda_i}$$

where the $P_i$ may be interpreted as the probabilities of symbol $i$ occurring.

The averaging inherent in this model implies that as the entropy increases, so the range of $\bar{\alpha}$ and $\bar{\beta}$ is reduced. We can quantify the bounds for such a variation for $\bar{\alpha}$ by invoking symmetry arguments. When we do this, we obtain a feasible region in the $H/\bar{\alpha}$ plane.

Within this plane, the value of $\alpha$ is constrained by two curves, I and II (see Fig. 2). Equation (9) gives the canonical form for these bounding curves, and (10) gives their corresponding $H$ and $\alpha$ values. Curve I follows from a minimization of $\alpha$ with increasing entropy and represents the important case of azimuthal symmetry. The minimum value is obtained by adding isotropic noise [the parameter $m$ in (9)] to the subspace orthogonal to the $\alpha = 0$ scattering mechanism. Curve II follows from a maximization of $\alpha$ with increasing entropy. In this case, two regions must be identified. The first is for low values of $m$, at which we can fill up the $\alpha = 0$ subspace with noise, and $\alpha_{\text{max}}$ stays at $90^\circ$. However, for $m < 0.5$ in (9), this subspace is filled eventually, and the noise starts to spill into the whole space.

$$[T]_I = \begin{cases} 1 & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 1 \end{cases} \quad 0 \leq m \leq 1$$

$$[T]_II = \begin{cases} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 2m - 1 & 0 & 0 \end{cases} \quad 0 \leq m \leq 0.5$$

$$H_I(m) = \frac{-1}{1 + 2m} \log_2 \left( \frac{m^{2m}}{(1 + 2m)^{2m+1}} \right)$$

$$H_{II}(m) = \begin{cases} \frac{-1}{1 + 2m} \log_2 \left( \frac{m^{2m}}{(1 + 2m)^{2m+1}} \right) \\ \frac{-1}{1 + 2m} \log_2 \left( \frac{m^{2m}}{(1 + 2m)^{2m+1}} \right) \end{cases}$$

$$\alpha_I = \frac{m\pi}{1 + 2m} \quad \alpha_{II} = \begin{cases} \frac{\pi}{2} \\ \frac{\pi}{1 + 2m} \end{cases}$$

When combined, we obtain the feasible region of $H/\alpha$ space shown in Fig. 2. We shall use this plane to represent radar data for the purposes of parametric inversion studies. However, first we consider the form of the scattering model used for inversion.

### III. PARTICLE SCATTERING MODEL

We now consider the case of scattering from a cloud of anisotropic particles. A schematic representation of a particle is shown in Fig. 3. We assume that only single scattering is significant, and that each particle in the cloud acts independently of its neighbors. Each particle has a scattering matrix in its characteristic coordinate system of the form

$$[S] = \begin{bmatrix} a & c \\ c & d \end{bmatrix} \quad a, d, c \in C$$

$$(II) = \frac{1}{2} \begin{bmatrix} (S_{HH} + S_{VV})(S_{HH} + S_{VV})^* & (S_{HH} + S_{VV})(S_{HH} - S_{VV})^* \\ (S_{HH} - S_{VV})(S_{HH} + S_{VV})^* & 2S_{HV}(S_{HH} + S_{VV})^* \end{bmatrix}$$

$$= \begin{bmatrix} e^{i\delta} \cos \alpha & e^{i\delta} \sin \alpha \\ -e^{-i\delta} \sin \alpha & e^{-i\delta} \cos \alpha \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} e^{-i\delta} \cos \alpha & -e^{-i\delta} \sin \alpha \\ e^{i\delta} \sin \alpha & e^{i\delta} \cos \alpha \\ 0 & 0 & 1 \end{bmatrix}$$

$$(II) = \begin{bmatrix} \frac{S_{HH} + S_{VV}(S_{HH} + S_{VV})^*}{2S_{HV}(S_{HH} + S_{VV})^*} & \frac{2(S_{HH} + S_{VV})S_{HV}^*}{4S_{HV}S_{HV}^*} \\ \frac{2(S_{HH} - S_{VV})S_{HV}^*}{2S_{HV}(S_{HH} - S_{VV})^*} & \frac{2(S_{HH} - S_{VV})S_{HV}^*}{4S_{HV}S_{HV}^*} \end{bmatrix}$$

$$(6)$$
where \(a\), \(c\), and \(d\) are complex scattering coefficients defined in terms of the particle polarisabilities \(\rho_1\), \(\rho_2\), \(\rho_3\), canting angle \(\theta\), and tilt angle \(\tau\) as shown in (12) \[11\]

\[
\begin{align*}
\alpha &= \rho_1 \sin^2\tau \sin^2\theta + \rho_2 \cos^2\theta + \rho_3 \cos^2\tau \sin^2\theta \\
\delta &= \rho_1 \sin^2\tau \cos^2\theta + \rho_2 \sin^2\theta + \rho_3 \cos^2\tau \cos^2\theta \\
\epsilon &= \rho_1 \sin^2\tau \sin\theta \cos\theta - \rho_2 \sin\theta \cos\theta + \rho_3 \cos^2\tau \sin\theta \cos\theta
\end{align*}
\] (12)

with

\[
\begin{align*}
\rho_i &= \frac{V}{4\pi(L_4 + L_4 + 1)} \\
L_4 + L_4 + L_4 &= 1 \\
L_4 : L_4 : L_4 &= \frac{1}{x_1} : \frac{1}{x_2} : \frac{1}{x_3}
\end{align*}
\] (13)

where \(V\) is the particle volume, and \(x_i\) are the particle dimensions. Of particular importance is the anisotropy \(A\), defined as the ratio of eigenvalues of \([S]\) and expressed in terms of the particle shape and material composition from (11) and (13) as

\[
A = \frac{L_4(c_r - 1) + 1}{L_4(c_r - 1) + 1}. \tag{14}
\]

Using (13) and assuming spheroidal shapes so that \(x_2 = x_3\), we can express this directly in terms of the particle shape ratio \(m = x_2 / x_1\) as

\[
A = \frac{m c_r + 2}{m + c_r + 1}
\]

\[
0 \leq m < \infty \quad \begin{cases} m < 1, & \text{Prolate spheroids} \\ m > 1, & \text{Oblate spheroids} \end{cases} \tag{15}
\]

From this, we see that if the dielectric constant is small \((\varepsilon_r \approx 1)\), then particle shape makes little difference to \(A\), and polarimetry is of limited applicability. However, if \(\varepsilon_r\) is large (as it is for many vegetation remote sensing problems in the microwave spectrum), then \(A\) becomes strongly dependent on the shape ratio of the particle. In this case, polarimetry becomes useful, and we can hope to invert the \([S]\) matrix data to obtain an estimate of particle shape.

We now assume that the particles in the cloud are identical in size but have a distribution of canting angle about the line of sight. For simplicity, we assume the tilt angle \(\tau = 0\) and a uniform distribution with mean \(\theta = (\theta_1 + \theta_2)/2\) and width \(\Delta\theta = \theta_2 - \theta_1\) for the canting angle, as shown in Fig. 4. Note that this is a more general model than that widely used in the radar literature \[12\], which normally supposes a uniform distribution over \(\pi\) radians. In this model, we can now include the case of aligned particles. Since natural vegetation often may be expected to have correlated orientation of leaves and scatterers, such an extension is important for attempting quantitative remote sensing inversion.

Using this model, we can now obtain an analytic form for the coherency matrix, as shown in (16)

\[
\langle T \rangle = \begin{bmatrix}
\epsilon & \frac{\rho \sin 2\theta \sin \phi}{2\Delta \theta} & \frac{\rho \sin 2\theta \sin \phi}{2\Delta \theta} \\
\frac{\rho \sin 2\theta \sin \phi}{2\Delta \theta} & \nu \frac{\theta + \frac{1}{2} \sin \theta |\phi|}{\theta_1} & \nu \frac{\theta - \frac{1}{2} \sin \theta |\phi|}{\theta_1} \\
\frac{\rho \sin 2\theta \sin \phi}{2\Delta \theta} & \nu \frac{\theta - \frac{1}{2} \sin \theta |\phi|}{\theta_1} & \nu \frac{\theta - \frac{1}{2} \sin \theta |\phi|}{\theta_1}
\end{bmatrix}
\] (16)

with

\[
\begin{align*}
\epsilon &= \frac{1}{2} |a + d|^2 \\
\mu &= \frac{1}{2} (a + d)(a^* - d^*) \tag{17} \\
\nu &= \frac{1}{2} |a - d|^2
\end{align*}
\]

We see that the structure in this matrix depends on the sum and differences of the particle scattering coefficients, as well as on the distribution of orientation angles.

The simplest case to consider is when the cloud is randomly distributed, in which case \(\theta_1 = 0\), \(\theta_2 = 180^\circ\), and \(T\) has the diagonal form shown in (18) \[\text{compare with } T_I \text{ in (9)}\]. Then, the eigenvalues of \(T\) are just its diagonal elements, and from these we can calculate the \(H/\alpha\) values for a random cloud.

If we consider particles with anisotropy \(A\) (15), then for \(A = 1\), we have either spheres or low dielectric material. For \(A < 1\), we have prolate particles (needles), and for \(A > 1\), we have oblate particles (discs)

\[
\langle T \rangle = \frac{1}{2} \begin{bmatrix}
2 \epsilon & 0 & 0 \\
0 & \nu & 0 \\
0 & 0 & \nu
\end{bmatrix}
\] (18)
Fig. 5 shows how the $H/\alpha$ values vary as a function of $A$. Note the following points.

- For $A = 1$, the entropy is zero and $\alpha$ is zero.
- Prolate particles yield a higher limiting entropy ($H = 0.95$) than oblate particles ($H = 0.62$).
- There is ambiguity as to the oblate/prolate shape of the particle for $H < 0.62$.
- If we plot the $H/\alpha$ values in a plane, they all lie along curve I in Fig. 2.

It follows from the last point, that we can consider curve I to be the locus of $H/\alpha$ values for a truly random distribution but varying shape. This will be important when we come to try and invert data to obtain estimates of the orientation distribution.

The next stage of analysis is to examine the case where the particle shape is fixed, but the cloud has a varying distribution of angles. Fig. 6 shows the $H/\alpha$ loci for this case, and we now assume that the tilt angle is distributed uniformly over $\pi$ radians. Here, we can see that when $A = 0$ (i.e. dipoles), the loci is approximately a straight line at constant $\alpha$ value ($45^\circ$ as expected from Table I). Hence, as the randomness of the distribution increases, it effects mainly the entropy $H$. In the limiting case of random distribution, we obtain a point on curve I at $H = 0.95$, as expected from Fig. 5.

As $A$ increases, so the loci remain of the same general form (i.e., lines parallel to the $H$ axis). In the limit $A = 1$, and the loci is a point at the origin of $H/\alpha$ space. If $A$ is further increased ($A > 1$), the loci turn back to points with increasing $H$ and $\alpha$. We conclude from this that the position of a $H/\alpha$ data point obtained from an estimate of $T$ for the scattering cloud, can be used to infer information about both particle shape and orientation distribution. Note that there is still the ambiguity between oblate and prolate particles but only for $H$ values below 0.62. This means that in Fig. 6, the loci below $H = 0.62$ are bivalued in $A$. For each loci, there is a value of $A > 1$ and a value $A < 1$.

The final stage of this analysis is to consider the extraction of the mean orientation angle $\theta$. When the entropy is zero, then $\theta$ may be estimated directly from the $[S]$ matrix data.

However, as the entropy increases, the estimate becomes biased by the averaging inherent in $(7)$. To investigate the effect of increasing entropy, we show the error between $\theta$ and the true mean value $2\theta$ in Fig. 7. We see that the error is a function of particle anisotropy $A$. For strongly anisotropic particles, the error can be significant.

For example, if we have no a priori idea about the shape of the particles, then Fig. 7 gives an indication of the error...
in mean orientation angle with entropy. However, if we can obtain an estimate of particle shape, then we can select the appropriate characteristic in Fig. 7 to compensate for the entropy bias and obtain an improved estimate of the mean canting angle. 

To illustrate the extraction of mean orientation angle, we consider the following examples. We consider a cloud of particles with shape anisotropy \( A = 0.1 \). In the first instance, such a cloud is oriented at a mean angle of 40° with a spread of ±40°. The normalized coherency matrix for this case is shown in (19). This matrix can be expressed in terms of its eigenvalues and eigenvectors and then estimates obtained for entropy, alpha and beta.

\[
T = \begin{bmatrix}
1.000 & -0.100 & -0.568 \\
-0.100 & 0.2962 & 0.014 \\
-0.568 & 0.014 & 0.3732 \\
\end{bmatrix}.
\]  

(19)

The entropy is \( H = 0.5 \) and the \( \beta \) estimate is 64°, so that the mean orientation is estimated at 32°. From Fig. 7 and using \( A = 0.1 \), we see that the expected bias error is around 8°, which can be used to correct this estimate.

In (20), we show the same particle cloud but now with a mean inclination of only 10°.

\[
T = \begin{bmatrix}
1.000 & -0.542 & -0.198 \\
-0.542 & 0.306 & 0.026 \\
-0.198 & 0.026 & 0.303 \\
\end{bmatrix}.
\]  

(20)

The entropy is again \( H = 0.5 \), but this time the estimate of \( \beta \) is 4°, corresponding to a mean inclination of 2°. Again, the corresponding bias is 8°, which can be corrected through knowledge of \( A \).

As an alternative to employing the mean \( \beta \), we considered estimating the mean cloud orientation, obtained as the orientation of the maximum eigenvector of \( T \) [as \( \beta_2 \) in (7)]. For cloud simulations, this showed less bias than the Bernoulli mean, and so it does not require the above correction. Future studies will address the best orientation estimator to use. However, for the tree data considered in this paper, the entropy is high and the orientation has a wide distribution (as later shown in Fig. 13). Hence, the mean orientation is not well defined, and we have ignored it in our experimental analysis.

IV. EXPERIMENTAL DATA

The experiment consisted of two 3-D SAR imaging measurements on a fir tree and a ficus tree, respectively. The measurement set-up used is shown in Fig. 8, while photographs of both trees are displayed in Fig. 9. These measurements were performed in a controlled environment, making use of the anechoic chamber of the European Microwave Signature Laboratory (EMSL) at JRC. In order to have stable measurement conditions, prior to and during the measurements the trees were watered regularly, and a constant artificial illumination replicating the sunlight’s spectrum was used to reduce the day–night effect. The fir tree used in this experiment was about 5 m high and 2.5 m wide. This tree presented quasihorizontal branches bearing typically 2–3 cm needles and showing a brush-like distribution. The branches constituted large horizontal planar surfaces at different levels in height. The top of the tree conformed to a young tree of the same species. The ficus tree was about 2.5 m high and 1 m wide. This tree showed 10–25-cm-long pointed leaves and many-twigged, slender branches.

The trees were put in a pot and then mounted on a rotating platform inside the anechoic chamber. The pot under the trees was covered with microwave pyramidal absorbers, to make sure that the measured backscatter did not include any contribution from the ground and the corresponding interaction with the tree. The measurement system used in this experiment was based on a network analyzer and operated in the stepped-frequency mode. The frequency band and the angular spans in azimuth and elevation used in the measurements are summarized in Table II. The acquired data in the frequency domain were empty room subtracted and gated in the time domain in order to isolate the response of the trees from the residual antennas coupling and the eventual spurious reflections in the chamber.
The reflectivity images of both trees were reconstructed by making use of a near-field focusing technique [13]. The spatial resolutions, achieved with the frequency bandwidth and the synthetic aperture lengths in elevation and azimuth indicated in Table II, are in the order of 6 cm in the vertical cross-range and ground-range directions, and 15 cm in the horizontal cross-range direction. A Kaiser-Bessel window has been applied over the frequency and the two aspect angles (in azimuth and elevation). This is enough to produce imagery in which the position of the scattering centers easily can be associated with a small volume within the tree structure. The 3-D images have been presented in [14] and are not repeated here for the sake of economy. As observed in that paper, for the fir tree, the differences in polarization were more evident in the top part of the tree, where there are almost no branches and the trunk is giving the main contribution to the backscattering. Consequently, the top part of the trunk is more visible in the $VV$ image. On the other hand, in the middle and bottom part of the tree, the tree architecture is more complex, and the differences in polarization are smaller. The backscattered power in $HV$ is comparable to those in $HH$ and $VV$, which indicates that the main scattering centers are associated primarily with the green, outer branches coated with needles. The ficus is more inhomogeneous and as a result, the reflectivity images can be described as a distribution of spots corresponding to leaves and branches.

Fig. 10 shows the spatial distribution of the $\alpha$ parameter, corresponding to slices of the images from the fir and ficus trees. The slices were taken at zero ground-range and zero cross-range. For the fir tree, it is evident that most values are close to 45$^\circ$, which reflects the contribution of the needle-shaped leaves and branches at different frequency scales. Moreover, we can observe some areas with $\alpha$ close to zero on the outer part of the branches. This indicates a surface-like behavior with no interaction with other tree parts. On the other hand, there are some small spots near the trunk with values close to 90$^\circ$, which are due to double-bounce reflections between the trunk and the branches. In the case of the ficus tree, there is a cluster of leaves in the near-range area that produce a clear, surface-like response. This response may be originated by leaves, which are normally oriented to the line of sight, thus pointing to the antenna. The other parts of the image show a dipole-like behavior that may be due to the scattering produced by the cylindrical branches where no leaves are pointing to the antennas.

In this study, we are interested mainly in the frequency variation of the $H/\alpha$ values for these volume scattering targets. If we consider the trees as inhomogeneous clouds of anisotropic scattering particles, then as the wavelength changes, we should become sensitive to different-shaped structures within the cloud. To investigate this further, we calculated the coherency matrix as a function of frequency by averaging overall azimuth and elevation angles. We are interested also in studying the dependence of the vegetation measurements on the incidence angles, since this could lead to a wrong estimation/interpretation of the involved scattering mechanisms. Then, we have calculated the coherency matrix as a function of the incidence elevation angle by averaging overall azimuth angles and frequencies. Note that we have computed the results only for the frequency band and angular span that intersect the measurements from both trees: 2–5.5 GHz and 39$^\circ$–51$^\circ$. This was done in this way to ease the comparison between the fir and ficus trees.

Using the $T$ matrices so obtained, we then calculated the $H/\alpha$ values. Fig. 11 shows the results obtained, plotted as points on the $H/\alpha$ plane. The varying parameters (frequency and incidence angle, respectively) are represented by the gray scale of the points, going from the black (lowest value) to the white (highest value).

It is obvious that the loci of the $H/\alpha$ points are different for these trees. Moreover, the trend is clearly the opposite as frequency increases. Entropy and $\alpha$ increase with frequency for the fir tree, while they decrease for the ficus tree. The result as a function of the incidence angle could be employed in classification procedures, since the points corresponding to the same tree are very close together but are well separated between tree types. We now use this data with the model of Section III to attempt inversion of the parameters $A$, $\theta$, and $\Delta \theta$, given $H/\alpha$. 
TABLE II
TREE MEASUREMENT PARAMETERS

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<thead>
<tr>
<th>Parameter</th>
<th>FIR TREE [Min, Max, Stop]</th>
<th>FICUS TREE [Min, Max, Stop]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (GHz)</td>
<td>1.5-5.5, 0.01125, 2.0-10.0, 0.01125</td>
<td>870-1020, 0.6, 46.0-70.0, 0.5</td>
</tr>
<tr>
<td>Tree Azimuth Rotation (\phi(\circ))</td>
<td>78.0-102.0, 0.6, 46.0-70.0, 0.5</td>
<td>28.0-52.0, 0.3, 39.0-51.0, 0.5</td>
</tr>
<tr>
<td>Antennas Looking Angle (\theta(\circ))</td>
<td>HH-HV-VH-VV</td>
<td>HH-HV-VH-VV</td>
</tr>
</tbody>
</table>

V. INVERSION ALGORITHM

From the average coherency matrix \(T\), we can obtain estimates of three main parameters: the entropy \(H\) from the eigenvalues, the average scattering mechanism \(\bar{\alpha}\), and the mean orientation angle \(\bar{\beta}\), all from the eigenvectors. It is of interest to see if we can use these estimates to obtain the three physical parameters of the particle cloud: the mean particle anisotropy \(A\), the mean orientation angle \(\theta\), and the width of the orientation distribution \(\Delta\theta\).

Using the model of Section III, we have an analytic, nonlinear mapping from the physical parameters into the observables. Formally, we may write

\[
\Omega = \begin{bmatrix} H \\ \bar{\alpha} \\ \bar{\beta} \end{bmatrix}, \quad \Gamma = \begin{bmatrix} A \\ \theta \\ \Delta\theta \end{bmatrix}, \quad \Omega = F(\Gamma) \tag{21}
\]

where the function \(F\) is given by (7), (8) and (16). If we denote the measured estimate of \(\Omega\) as \(\bar{\Omega}\), then the inversion procedure can be stated formally as

\[
\bar{\Gamma} = F^{-1}(\bar{\Omega}) \quad F^{-1} = \min \; \text{norm}(\bar{\Omega} - F(\Gamma)). \tag{22}
\]

In practical terms, the inversion proceeds in the following manner.

1) From an \(H/\alpha\) point, obtain an estimate of the two parameters \(A/\Delta\theta\) as in Fig. 6.
2) Use the estimated \(A\) value and \(H\) to obtain a corrected \(\beta\) value.
3) From the new \(\beta\), obtain an estimate of \(\theta\).

The most difficult stage is the first, in which a two-parameter optimization must be employed. We have used a straightforward simplex numerical optimization method, as it requires no gradient information and is easy to implement.

Figs. 12 and 13 show the result of applying this inversion scheme to the experimental data of Fig. 11. We show the estimates as a function of frequency for the parameters \(A\) and \(\Delta\theta\). Note that, regarding to the ambiguity cited in Section III, we have selected the prolate parameters \(A < 1\) for the fir tree and the oblate parameters \(A > 1\) for the ficus tree. This is based also on the loci occupied by the points in the \(H/\alpha\) plane for both cases (\(H > 0.62\) for the fir tree and \(H < 0.62\) for the ficus tree) and on the \textit{a priori} knowledge about the physical characteristics of the trees. Even if we did not know whether it was oblate or prolate, the shape parameters obtained for the two trees would be very different, because they are well separated in \(H/\alpha\). Hence, we have a good classification and have a two-valued shape parameter only for low entropy (\(H < 0.62\)) vegetation.
The next step is the physical interpretation of the results from Figs. 12 and 13. Note the following comments concerning the results from the fir tree.

- For frequencies below 3.5 GHz, the backscatter appears to be dominated by particles of fixed anisotropy (around $A = 0.12$). This fact gives rise to a lower entropy than at higher frequencies, reflecting the fact that scattering here is dominated by larger-scale branch and trunk structures.
- For frequencies above 3.5 GHz, the backscatter mechanism changes and becomes dominated by scatterers of different shape at different wavelength scales. In this transition region, the particles become more and more
anisotropic (more like needles), and the width of the angular distribution increases. This is consistent with scattering from smaller-scale branches and needles.

- At the shortest wavelength, the scattering is dominated by the smallest-scale, needle-like structures on the tree, which have essentially a random distribution (a random cloud of dipoles) and highest entropy (0.95).

As expected, the results from the fir tree are consistent with the particle model employed in the inversion, since the structure of the tree (trunk-branches-leaves) is regularly scaled as a function of the frequency. As a result, different parts of the trunk have been identified in the model at different frequencies, following a continuous change. However, this is not the case for the ficus tree. This kind of tree has two completely different components in its morphology: leaves and branches. As stated previously, leaves are fairly elliptical discs, and branches are cylindrical. We can see in Fig. 12 that for lower frequencies (less than 2.5 GHz), the particles present high values of $\mathcal{A}$ corresponding to clouds of discs, which are the dominant mechanism in this case. At higher frequencies, the discs are more directive and only some of them are pointing to the receiver, due to their random orientation. Thus, their response is averaged with the response from branches. Consequently,
the $A$ values shown in the results are not related directly to any single scattering mechanism from the ficus tree. Note that both fir tree and ficus tree were isolated (from the scattering viewpoint), and therefore, the effect of the ground and the ground–tree interactions is not present in these results.

The width of angular distribution behaves in a similar way for both trees. This can be explained by looking at the $H/\alpha$ loci in Fig. 11. The data are close to curve I at the border of the feasible region, and this curve corresponds to azimuthal symmetry, so there is no dependence on orientation at all, and both distributions are very wide. It is the $H/\alpha$ values that provide the discrimination between these data sets, not the mean orientation.

According to the variability present in Fig. 13, it is important to state that it is due to the randomness of the input data and not to any potential sensitivity of the inversion to small errors in the input data. The fairly wide separation between loci in Fig. 6 indicates that this technique is basically robust, because small changes in the input data will give small changes in the inverted parameters. However, for our data, the entropy is high and so their statistical variability is high, requiring a large number of looks for good estimates of the eigenvalues and hence, the $H/\alpha$. Details of the variance of the estimates versus the number of looks can be obtained by assuming Gaussian statistics and employing the complex Wishart distribution [15].

So far, a comparison of the estimated particle anisotropy with the actual anisotropy (ground truth data) has not been tried. This exercise will be carried out in the future as part of an ongoing, detailed, morphological structure analysis of the fir tree [16], [17].

These results indicate that such an inverse model can be employed usefully for the study of canopy scattering effects and for the inversion of radar data for vegetation and forestry classification problems. Although such shape and angle distributions are built into forward scattering models like vector radiative transfer [18], this is the first attempt known to the authors to extract these parameters from radar data in an inversion process. On the other hand, the mean orientation already has been applied to the extraction of surface slopes in [19], where the polarization signature was used instead of the eigenvector estimation. As stated before, in the experimental examples presented in this paper, the entropy is so high that the orientation distributions are wide and, therefore, the mean orientation is not a useful parameter. This is because we have carried out extensive averaging over azimuth and elevation to produce the estimates. We expect $\beta$ to be more useful for vegetation rather than forest applications, and future work will address this issue.

The results also show that the polarimetric signature is sensitive to the structure of the sample as well as to its volume. This has important consequences for the use of multifrequency polarimetric SAR systems for vegetation studies.

VI. CONCLUSION

In this paper, we have shown how an eigenvalue analysis of the average backscatter coherency matrix may be employed (with a simple model of particle scattering) to understand the physical basis of the radar observables in a clearer way than is obtained by looking at a simple polarimetric ratio such as $HH/VV$. In particular, we have shown that the entropy/alpha plane is a useful representation of the average properties of the data.

We have shown that in this plane, effects due to particle shape and orientation distributions are well separated and hence, we can employ this method in a robust inversion procedure to estimate the physical parameters of a cloud from experimental radar data. We have illustrated this algorithm by applying it to broad-band experimental data collected for a fir tree and a ficus tree at the EMSL. The results show a wavelength scale dependence of the shape and distribution of scatterers, which reflects the complex volume scattering nature of such problems. These two trees present different responses in the entropy/alpha plane and opposite trends as a function of the frequency, as expected from their morphology. We conclude that such an algorithm has the potential to be applied to field radar data for practical remote sensing problems, in which vegetation and canopy classification studies are important.

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REFERENCES


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