Extension of the chirp scaling algorithm to 3-D near-field wideband radar imaging

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Abstract: The authors’ objective is to develop an efficient algorithm to focus two-dimensional and three-dimensional SAR images under the following extreme conditions: large coherent integration angles and high bandwidth-to-centre-frequency ratios. These conditions constitute a common situation in wideband high-resolution SAR configurations, as in anechoic chambers and in ground-based systems. The algorithm presented is an extension of the well known chirp scaling algorithm (CSA). First, the original 2-D CSA is generalised by extending the formulation from data obtained with a linear aperture to data measured by a planar synthetic aperture. Moreover, a compact generalised formula of the impulse response function in the three-dimensional frequency domain is presented. These extensions enable the development of a 3-D algorithm under the aforementioned conditions. Finally, the algorithm has been implemented and tested with satisfying results.

1 Introduction

Synthetic aperture radar (SAR) [1] is used for the generation of reflectivity images of scenes illuminated by microwave pulsed signals. The formation of such images requires the coherent combination of the echoes recorded by the SAR sensor at uniformly spaced positions in order to focus all the targets properly. In this way, the final azimuth resolution in the images is similar to that obtained with a large antenna. The problem of focusing SAR data is generally approached by considering the signal received from a single point scatterer. Hence, the system is fully described by a model of its impulse response function (IRF). The IRF is useful for formulating the focusing procedure of real scenes, which are subsequently modelled by a combination or superposition of point scatterers.

Focusing algorithms are meant to correct for the effects of range curvature and the variation of this curvature over range. These effects are easily and accurately compensated for in conventional airborne and spaceborne systems. The design of the focusing procedure, however, is more complicated under either or both of the following extreme conditions:

(i) large coherent integration angle
(ii) low centre-frequency-to-bandwidth ratio.

This situation will be referred to hereafter as near-field wideband conditions. Such conditions appear when working with high-resolution radars and, in particular, with ground-based systems and anechoic chambers. Some of the applications of these systems are unexploded ordnance (UXO) and buried-object detection and classification, as well as precise characterisation of man-made and natural targets (e.g. vegetation, soil), based on their reflectivity.

A similar problem is common in high-resolution synthetic aperture sonar (SAS). SAS systems operate in the kilohertz region and present a low centre-frequency-to-bandwidth ratio. Note that this ratio is denoted as quality-factor ($Q$) in these systems [2]. Therefore, the image processing requirements of SAR (under the conditions previously mentioned) and SAS are equivalent. The main purposes of SAS are sea-bed and sub-bed imaging, as well as the detection of buried, proud bottom (non-buried) and tethered mines.

An accurate procedure for achieving excellent precision in near-field wideband conditions is the so-called range migration algorithm (RMA), or $\omega-\kappa$ [3, 4]. The drawback of this technique, however, is its interpolation stage, which implies a high computational cost and which slows down the generating of the image.

An appropriate alternative procedure is the chirp-scaling algorithm (CSA) [5-7], which avoids the interpolation, as it only requires Fourier transforms and complex products. It is therefore more efficient than the RMA. The original CSA produces high-quality images when it is applied to conventional spaceborne and airborne SAR systems. The generation of accurate images by the original CSA in near-field wideband conditions, however, is limited by the approximations assumed in its formulation, even for targets located at the reference range for which the algorithm is tuned.

The key point is that standard CSA formulations only provide terms of up to the second or third order of the Taylor series expansion of the IRF phase in the range–frequency/azimuth–frequency domain [6], and higher-order terms are neither considered nor presented. The inclusion of additional terms (up to an arbitrary $n$th-order) makes it possible to maintain the efficiency of the methodwithout sacrificing accuracy.

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and improve on its accuracy considerably in near-field wideband conditions. The main contributions of this study, therefore, are:

- extension of the original CSA to the 3-D case of a planar aperture;
- general expression of the IRF phase for chirped signals in the 3-D frequency domain, that enables the improvement of radar and sonar imaging techniques under near-field wideband conditions.

2 Formulation and flow chart

Consider the measurement set-up shown in Fig. 1. The antenna, which is located at \((x_a, y_a, z_a)\) co-ordinates, synthetises a planar aperture of size \(L_x \times L_z\) at a distance \(y_{ac}\) from the scene centre. The aperture is parallel to the XZ plane and is centred at \((0, y_{ac}, 0)\). Since we are dealing with a planar aperture (not a linear aperture as in 2-D imaging), we define the azimuth (or cross-range) plane with the co-ordinates \(x\) and \(z\). Note that the formulation derived in this paper is suited to the broadside configuration presented in Fig. 1. The range co-ordinate of a point in the scene, \(r\), is the orthogonal distance from the aperture plane. In this broadside configuration, the range co-ordinate can be expressed as \(r = y_{ac} - y\). We suppose a chirp modulation radar with a chirp rate \(\Gamma\) and a centre frequency \(f_c\).

The formulation of the 3-D imaging algorithm has been derived by following the same general procedure as in the 2-D case, which is described in [1]. The main differences in the new algorithm, with respect to the original one, are:

(i) one-dimensional (1-D) Fourier transforms along the cross-range co-ordinate are obviously substituted by 2-D Fourier transforms over the cross-range plane;

(ii) impulse response function (IRF) presents a new expression that must be obtained to formulate the focusing steps.

The analytical expression of the IRF in the frequency domain can be derived in a three-stage procedure [1, 5].

First, by assuming that there is a point scatterer located at \((x, y, z)\) with a reflectivity of one, the analytical expression of the baseband impulse response \(s(\tau, x_a, z_a; r)\) can be derived by

\[
\begin{align*}
    s(\tau, x_a, z_a; r) &= s_0(x_a, z_a) \cdot s_p(\frac{\tau}{c} \cdot R(x_a, z_a; r)) \\
    &\quad \times \exp \left( -j\pi \Gamma \left( \frac{\tau}{c} - \frac{2}{c} R(x_a, z_a; r) \right)^2 \right) \\
    &\quad \times \exp \left( -j\frac{4\pi}{c^2} R(x_a, z_a; r) \right)
\end{align*}
\]

where \(\tau\) is time in the range direction (short time). The function \(s_0(\cdot)\) is the azimuth aperture-weighting pattern, \(s_p(\cdot)\) is the transmitted signal envelope and \(R(x_a, z_a; r)\) is the instantaneous distance between the sensor and the point scatterer. The constant \(c\) is the velocity of the light and \(\lambda_c\) denotes the wavelength at the central frequency of the radar sensor.

The formulation begins with the range Fourier transform of this signal impulse response by applying the method of stationary phase (MSP), followed by an azimuth Fourier transform of the resulting expression (again by MSP). Thus, the following expression of the point scatterer phase response in the frequency domain is obtained:

\[
\Phi_{IRF}(k_x, k_z; r) = -k_x x - k_z z + r_0 k_x + \frac{\lambda_c^2}{16\pi^2} k_x^2 + \frac{4\pi r}{\lambda_c} A_{xz}(k_x, k_z) \\
\times \left( 1 + \frac{\lambda_c}{A_{xz}(k_x, k_z) \cdot 2\pi k_r + \frac{\lambda_c^2}{4\pi^2 k_r^2}} \right)^{1/2}
\]

where

\[
A_{xz}(k_x, k_z) = \left( 1 - \left( \frac{\lambda_c}{4\pi k_r} \right)^2 - \left( \frac{\lambda_c}{4\pi k_r} \right)^2 \right)^{1/2}
\]

The analysis continues by replacing the formula in (2) by its Taylor series expansion. In (2), the reference range is usually selected as \(r_0 = y_{ac}\) (the range co-ordinate at the scene centre).

To do so [1], in the last term in (2) we assume that the fractions in brackets are small relative to one. Then, the following expression:

\[
f(k_x, k_z; r) = \left( 1 + \frac{\lambda_c}{A_{xz}(k_x, k_z) \cdot 2\pi k_r + \frac{\lambda_c^2}{4\pi^2 k_r^2}} \right)^{1/2}
\]

can be replaced by its Taylor series expansion

\[
\left( 1 + \frac{\lambda_c}{A_{xz}(k_x, k_z) \cdot 2\pi k_r + \frac{\lambda_c^2}{4\pi^2 k_r^2}} \right)^{1/2} \approx \sum_{i=0}^{n} \frac{1}{n!} \frac{\partial^i f(0, k_x, k_z; r)}{\partial k_r^i} k_r^i
\]

\[
= \frac{1}{A_{xz}(k_x, k_z)} \cdot A_{xz}(k_x, k_z) \cdot \frac{\lambda_c^2}{4\pi^2} G(k_x, k_z) \cdot k_r^i
\]

\[
G(k_x, k_z) = \sum_{p=\text{cell}(i/2)}^{i} \left( \frac{2p-3}{i-3} \right) \cdot \left( \frac{i-3}{p-3} \right) \times \left( \frac{(-1)^p}{(p-2)} \cdot A_{xz}^{i-p-1}(k_x, k_z) \right)
\]

where

The operator \(\text{cell}(i/2)\) rounds off \(i/2\) to the nearest integer towards infinity.

By substituting (5) in (2), and by grouping terms together, we obtain the \(n\)-th order expression of the Taylor
series expansion for the point scatterer phase response in the frequency domain

\[
\Phi_{IRF}(k_x, k_z, r) = \sum_{i=0}^{n} \frac{1}{n!} \frac{\partial^n \Phi_{IRF}(0, k_x, k_z, r)}{\partial k_x^i} \cdot k_x^i = \sum_{i=0}^{n} \phi_{k_x}(k_z, r) \cdot k_x^i
\]

\[
\approx -k_x x - k_z z - \frac{4 \pi}{c} r A_{2x}(k_z, k_z) + \left( r_0 - \frac{r}{A_{2z}(k_z, k_z)} \right) k_z + \frac{c^2}{16 \pi} F_3(k_x, k_z, r) - \sum_{i=3}^{n} \frac{r}{2i-2} \left( \frac{\lambda}{4 \pi} \right)^{i-1} G_i(k_x, k_z) \cdot k_x^i
\]

where

\[
F_3(k_x, k_z, r) = \frac{1}{1 + \Gamma(2 \lambda/c)^2 \left[ \left( \frac{(\lambda/4 \pi k_z)^2}{1 - [(\lambda/4 \pi k_z)^2 - (\lambda/4 \pi k_x)^2]^{1/2}} \right) \right]}
\]

We can discard cubic and higher-order terms in the case of a narrow angle and a moderate relative bandwidth, but in near-field wideband conditions we have to consider terms of higher orders than squared to correct for the extra range curvature produced by such a configuration. Indeed, this expression enables us to control the approximation error in extreme conditions by a phase function multiplier in the range–frequency/azimuth–frequency domain. The results will prove the usefulness of this formula in the generation of images with excellent radiometric and geometric accuracy.

The near-field wideband algorithm consists of transforming the chirped SAR data into the frequency domain with a fast Fourier transform (FFT) and introducing a new multiplier phase function that compensates for the required terms in the Taylor expansion at a reference range. The required image accuracy and the importance of the near-field wideband situation determine the highest term in this phase function. Since these high-order terms are not coupled with the rest of the phase functions, the remaining steps are equivalent to the original CSA. The flow chart of the algorithm is displayed in Fig. 2.

In the case of stepped-frequency radars, the signal data are in the range–frequency domain and they are not chirped. The data must be first transformed to the 3-D frequency domain, and then a chirp operation and the near-field wideband correction can be performed at the same complex product.

The phase correction implemented in this algorithm, which is needed to compensate properly the phase curvature produced by the near-field wideband situation, is based on the Taylor series expansion of the last term of

Fig. 2 Flow chart of the near-field wideband 3-D CSA for chirped and stepped-frequency radar data
the phase in (2). There exists an alternative phase correction that has been implemented and does not make use of this series expansion. It consists in applying directly the phase correction of the last term in (2), using the reference range. In a second stage, the useful terms for the rest of the algorithm (first- and second-order) are rearranged. By this way, terms of all orders are considered in a single expression. This alternative implementation of the imaging algorithm is more straightforward. Anyway, the algorithm based on the series expansion has been used in the simulation results to illustrate the importance of the additional terms incorporated in near-field wideband cases.

3 Results

Simulations were carried out to verify the improvement obtained by cancelling higher orders of the phase in the frequency domain than in the original CSA. For the sake of clarity, we started by simulating a linear aperture and a single point scatterer at the centre of the scene. The length of the aperture was 2 m, and was centred at 2 m from the target. The reference range was chosen \( r_0 = 2 \) m in the processing. The frequency ranged from 2 to 6 GHz. Note that this configuration produces an extreme integration angle (62°) and quite a low centre-frequency-to-bandwidth ratio \( (f_c/B = 1) \). For comparison purposes, some two-dimensional images have been obtained with the original CSA and the near-field wideband CSA, including extra terms from the third to the seventh order.

Fig. 3 presents two orthogonal sections of the amplitude image, intersecting at the scatterer position. In the section at a constant range, shown in Fig. 3a, we observe that the width of the main lobe gets smaller when we increase the maximum order in the series expansion. This resolution enhancement is achieved because, when we increase the approximation order, a larger bandwidth is correctly processed in the azimuth–frequency domain, thus improving the resolution of the image in the azimuth direction. Furthermore, the cut at a constant azimuth, in Fig. 3b, shows that the first secondary lobes exhibit different levels. As the order increases, this difference begins to disappear. We can also deduce, from such results, that an optimum order can be found for any configuration. Evidently, this optimum order depends on the parameters that define a near-field wideband situation: coherent integration angle and centre-frequency-to-bandwidth ratio.

It is important to note in Fig. 3b that the target response obtained by the original CSA is shifted in range. Again, this offset is produced by an inaccurate phase compensation when there is a near-field case, because the Doppler bandwidth is larger that in narrow-angle situations, and the second-order approximation assumed in this algorithm is not sufficient to correct for the exact phase history. For example, if we increase the distance from target to the aperture up to 10 m (instead of 2 m), the range offset in the image obtained by the original CSA is very small.

A second simulation was performed using the same SAR configuration with one point scatterer at the reference range and at zero azimuth. Fig. 4 allows us to compare the exact phase response of the RMA with the phase response of the CSA for different orders at the position of the scatterer in the range–image/azimuth–frequency domain. This test confirms the reduction of the phase error as the processed order increases. As the order is increased, the response of the CSA tends towards the exact response obtained with the RMA. We also find here that a larger bandwidth in the azimuth frequency domain can be processed, as we have commented before.

An additional simulation was carried out, under the same conditions, in order to quantify the dynamic range of the focusing algorithm. In this case, the scene was
formed by nine point scatterers with reflectivity values of between 0 and ~80 dBsm. A Blackman–Harris window with a peak side-lobe ratio of ~92 dB has been employed. The results demonstrate that the dynamic range of the algorithm is greater than 70 dB. By comparing the values of reconstructed and actual reflectivities, the maximum error is lower than 1 dB. Table 1 illustrates the results obtained when applying the near-field wideband CSA up to the seventh order. In practice, the dynamic range will be limited by the presence of noise.

The performance of this algorithm was also tested experimentally by employing data from the outdoor linear SAR system, called LISA, developed at the European Microwave Signature Laboratory (EMSL) at JRC Ispra, Italy. This system is based on a stepped frequency radar equipped with a 2-D positioning system. The target consisted of a 3-D arrangement of eight metallic spheres of a diameter of 7.62 cm. The dimensions of the synthetic aperture were 1 m × 1 m. The frequency ranged from 15.5 to 17.5 GHz. The distance from the centre of the aperture to the scene centre was 2.3 m. The plane of the aperture was tilted 14° from the vertical direction (see Fig. 5). This set-up corresponds to an integration angle of 53° and a ratio \( \frac{f_c}{B} = 8.25 \). The theoretical resolutions are, in this case, 2 cm in the horizontal and vertical cross-range directions and 7.5 cm in the ground-range direction.

Fig. 6 shows several slices of the reconstructed 3-D image. The displayed dynamic range is 20 dB. As expected, the reflectivity at the positions of the spheres is about ~23.4 dBsm, corresponding to RCS given by the physical optics approximation. The measured spatial resolutions are in agreement with the expected ones. Note that the reflectivity peaks of the spheres closer to the antennas are narrower because the effective synthetic aperture is larger in the near range. On the other hand, the spheres have a diameter of about four wavelengths and therefore they are not ideal point scatterers. As a result, a minor degradation or defocusing must be expected.

It is important to note that this data set was already used [4] to validate the 3-D RMA. Consequently, we can now perform a comparison between these two techniques. In principle, the near-field wideband CSA is not as accurate as the RMA, due to the approximations assumed in the

### Table 1: Reflectivity values obtained in testing the dynamic range of our approach

<table>
<thead>
<tr>
<th>Position: (x, y) (cm)</th>
<th>Scene reflectivity (dBsm)</th>
<th>Image reflectivity (dBsm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(−40, 40)</td>
<td>−80.0</td>
<td>−80.14</td>
</tr>
<tr>
<td>(−40, 0)</td>
<td>−10.0</td>
<td>−10.43</td>
</tr>
<tr>
<td>(−40, 40)</td>
<td>−20.0</td>
<td>−20.72</td>
</tr>
<tr>
<td>(0, −40)</td>
<td>−70.0</td>
<td>−69.69</td>
</tr>
<tr>
<td>(0, 0)</td>
<td>0.0</td>
<td>0.00</td>
</tr>
<tr>
<td>(0, 40)</td>
<td>−30.0</td>
<td>−29.97</td>
</tr>
<tr>
<td>(40, −40)</td>
<td>−60.0</td>
<td>−60.38</td>
</tr>
<tr>
<td>(40, 0)</td>
<td>−50.0</td>
<td>−50.46</td>
</tr>
<tr>
<td>(40, 40)</td>
<td>−40.0</td>
<td>−40.79</td>
</tr>
</tbody>
</table>

![Fig. 5](image-url)  
**Fig. 5** Measurement set-up use in the experimental validation with a 3-D array of eight spheres (not to scale)  
*a* Top view  
*b* Side view  

![Fig. 6](image-url)  
**Fig. 6** Slices of the reconstructed 3-D image  
For front view, \( Y = 16, 0, -16 \) cm (top to bottom)  
For side view, \( X = 16, 0, -16 \) cm (top to bottom)
formulation. However, even under the extreme conditions presented in these tests, its performance is accurate enough for most applications. With respect to the computation time, we have run both algorithms on the same platform for the sake of comparison. The 3-D RMA took 82 seconds to process the data, and the near-field wideband 3-D CSA took 65 seconds. This time reduction is provided by the substitution of the Stolt interpolation by Fourier transforms and complex products.

4 Conclusions

An efficient algorithm to focus three-dimensional SAR images has been formulated and implemented. This technique is an extension of the chirp scaling algorithm. In the formulation of this procedure, an \( n \)th-order general expression of the phase impulse response of a synthetic aperture system in the three-dimensional frequency domain has been obtained. Thanks to this contribution, additional terms have been included in the CSA to reconstruct more accurate images in near-field wideband configurations. The number of extra terms to be considered depends on the particular configuration and the geometry of the measurement, and it can be freely selected in the algorithm because an \( n \)th-order expression is now available.

The algorithm has been implemented and tested with simulations and experiments. When compared with the original CSA, an evident improvement has been demonstrated by observing the shape and magnitude of the reconstructed reflectivity of a point scatterer. Numerical simulations have also shown that the dynamic range is better than 70 dB, even under extreme conditions.

This algorithm is suited to ground-based systems and anechoic chambers, as well as airborne and spaceborne systems with high bandwidths because they require motion compensation and other auxiliary processing which can be implemented easily in the CSA, but not in \( \omega-k \). Furthermore, the algorithm can be combined with other extensions of the CSA previously proposed by other authors [6, 7].

Finally, it is important to note that this extension of the algorithm can be applied to any synthetic aperture system, such as sonar and radar.

5 Acknowledgments

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6 References


